AE3030 Notes past Page 75

3Reynold's number: The relation of inertial forces to viscous forces.

Rho*L*Vinf / mu

Mu / rho is v

U is dynamic viscosity and v is kinematic viscosity.

What do we do after solving BL equations?

We are interested ins sskin friction and other relevant quantities.

For example Boundary Layer Thickness d(x)

How thick is it at x?

The y-location where u/ue (ue is the edge velocity) reaches 0.99 percent of the free stream velocity.

Ue is velocity that you've solved for w/o viscous effects. (There is a diagram on page 76)

Displaceement thickness d*(x)

*Everywhere on a streamline, vecloity is tangential.

So displacement thickness is how much the streamline has to displace to account fofr the same mass flow rate with a BL.

Missing mass flow = rho_eu_ed*

D* is the integral from 0 to infinity of (1 – rho*u / rho_eu_e) dy

Momentum thickness theta(x)

Theta = integral from 0 to infinity of rhou / rho_e_u_e [1 – u/ue] dy

Both displacement thickness and momentum thickness have units of distance.

Momentum thickness is momentum loss within the boundary later and is a result of reduced velocities within the boundary layer.

It is defined as the integral above (dunno why I wrote this)

Shape Factor H

The quantity is defined by d*/theta and has no units.

For laminar flows, H varies between 2 and 3. In turbuluent flows, H is usually 1.5 and 2. This is useful for figuring out what kind of flow you have.

Wall shear Stress T_wall

T_wall = T_w = mu(du/.dy) at wall

The derivative of u is computed numerically for a linear profile, du/dy is constant so T_w is constant. (this is page 78)

Shear stress is uaully non-dimensionalized – can be non-dimmed in different ways

C_f = T_w / (1.2rho_eu_e^2) -- this is for flat plates

To get Dragm you need to integrate C_f over the surface area.

D = integral of T_wdx

C_d = integral_fdx (non-dim)

Drag coefficiecient

C_d = D/ 1/2rhoinf*v^2Cc

If you integrate fcF to get c_d, you are only getting viscous drag, no pressure drag.

Some exact solutions

-fif we’re looking at a flat plate, celoity is not changing.

Continuity divv = 0

Dp/dx = 0. THIS velity is constant.

Blasius Flow

Blasius was Prandtl’s PhD student.

Masius’s Solution is what’s called a stream function.

U = dpsi/dy

V = -dspy/dx

U velocity is a partial stream fxn w.r.t y. If you put it into continuity, it works like this:

D^2psi/dydx + d^spy/dxdy = 0.

The stream function automatically satisfies continuity , so all we caere about is u-momentum. But we do’t know what psy is. But we know that psy has to be in m^2/s.

Transformation

Blasius made up eta.

Eta = ½ sqrt uinf/v_x) *y) this is a non-dimensional parameter

Psy(x,y,v) = sqrt(vu_infx) * f(eta)

Blasius equation is:

F_etaetaeta + f*f_etaeta = 0

Basically it's a 3rd order ODE for f/

3rd order, so we have 3 boundary conditions.

At the wall, we know that u = 0 v = 0.

At the edge of the boundary layer we know that u = u_e.

So all these were solved by Blasius: Boundary layer thickness, displacement thickness, momentum thickness, skin friction coefficient, Cd. Etc. (page 80)

INTRODUCTION TO TURBULENCE

Until now, we have looked at steady flows

-Vortex shedding

-Karman flow around a cylinder

These flows are unsteady but periodic.

Reynolds

He did experiments with varying fluid velocity in a pipe and dye. (Laminar flow) vs turbulent flow.

What is it that causes instabilities?

Instabilities grow because of some inhgerent instability within the fluid.

In boundary layers…

Transition (i.e. stable flow to unstable flow)

--reynold;s unmber is high enough

-External disturbances which cause initial disturbances happen

-adverse boundary layer gradient which causes separated flow with an inflection point

-wall roughesss, which acts both as a disturbance and as a factor that promotes reversed flow

-heating the wall causes instabilities to grow and vice versa

---

Frreestream disturbances

-disturbances in the outside flow can induce disturbances(There is a good diagram about adverse pressure gradient on page 100)

Also a good explanation about roughness and wall heating/cooling effects.

Cooling stabilizes the flow. If we cool the walls, you can have laminar flow for longer.

Airfoil designers try to postpone adverse pressure gradients.

The want to delay separation./transition

-laminar flows are desirable

In gliders, wind turbines and long range/long endurance aircraft.

Michel's method ofr transition preditction:

R_ex = u_e*/v

Re_theta = u_e*theta/V

These are VERY rough estimates

Turbuluent flow

-very brief discussion

-only how to modify navier-stokes

-better to go to grad school to fully understand this

Turbulent flow has eddiesthat are big as the diameter of the pipe and then smaller, smaller, smaller, etc.

Features of turbuluent flow:

-irregular fl;uctuations in species concentration, temperature, and velocity

-Turbulent mixing is very important in many applications because it is 100, 1000, or even a million times more powerful than molecular mixing.

Splitting a flow int oa mean flow and fluctuations

(there is a diagram on page 102)

Decompression of Flow properties

The variables u,v,w,and p all now become (U, V, W, P) + (u', v', w', and p')

Contribution of Velocity

Fluctuations to Transport Momentum….

-consider a control volume

Momentum rate crossing the face is rho (U + u')^2 deltaA.

We can directly computer U and u' both – it is called a Direct Numerical Simulation.

We can do it on a Very small timescale at VERY VERY Small scales and it's VERY EXPENSIVE

Then we can do Time Averaged Momentum Rate

RANS __> Reynolds Averaged Navier-Stokes

(this derivation is on page 103(go all the way through to page 104

That's it for RANS

Prandtly-Turbuluent flow over a flat plate

Boussinesq Hypothesis

-reynold's stresses can be written as a product of "eddy viscosity" and velocity.

Different regions of turbuluent boundary layers – viscous sublayer – viscosity dominates so eddies cannot form. No transport.

-Buffer region

-Inertial sublayer – eddy transport is dominant

-defect layer

Viscous Sublayer

-onlymolecular transport is happening – it's very close to the wall.

(stopped taking notes here – pagg 106)

LOW SPEED AERODYNAMICS

An irrotational flow is when vorticity = 0

Outside the boundary later, flow is mostly irrotational

Stream function psi:

Derivation on how the stream function is incompressible and irrotational on page 108

What is the physical significants of psi?

Page 108 – 109.

NEXT: Velocity potential.

We're using phi for velocity potential. Potential functions are irrotational only, but we are not limited to 2D flows.

Velocity potential satisfies the Laplacian.

Potential function is constant.

Derivation on page 111.

RELATION BETWEEN STREAMLINES AND POTENTIAL LINES:

Constant streamlines and constant potential lines are orthogonal to each other. (proof on page 111)

Polar Coordinates of Stream Function and potential function (page 112)

A quick word about Laplace's equation – it's linear and 2nd order PDE – the sum of solutions is also a solution (that's what linear means)

-for irrotational, incompressible flow, both the stream fxn and the velocity potential satisfy laplace's equation.

Circulation

We have a flow field and a random curve

Circulation is the negative of the line integral of the velocity integral dotted with the line differential. (see equation on the bottom of page 112). The negative sign is b/c of aero convention.

Circulation depends on the curve you choose.

Circulation over airfoils…

LIFT is generated by…

-once flow hits, flow speeds up

-Kutta condition (flow needs to be zero./point in the same direction)

-air speeds up to meet kutta condition

BUILDING BLOCKS

i) Uniform flow (derivation on page 114
ii) Source/Sink (derivation on page 115 to 117)

A combination of building blocks is on page 118

iii) Doublets (source-sink pairs) (see pages 120-121)

Non-lifting flow over a circular cylinder 128-129

Non-kifting flow over a circular cylinder.

If you have uniform flow in x + a doublet, you get

Psi = u_inf*y = u_inf*rsintheta

For a doublet

And

Psi = -mu/spi *sintheta/r

Add the 2 functions, you get…

Psi = u_inf*rsintheta – mu/2psintheta/r

And psi = u_infrsintheta (1 – mu.spiu_inf) 1/ r^2) the mu being the doublet strength in this case.

In short, you have psi = u_inf*r*sintheta[1 – R^2/r^2] Where R^2 = constant, and is equal to mu / 2piu_inf.

So you have V_r = 1/r * dspi/dtheta ..the rs cancel out, costheta coes to 0 where theta equals pi/2 and 3pi/2

But also goes to zero when R^2/r^2 = 1 = R = r

And now we have to ignore when r is less than R because it's non-physical.

V_theta = -dpsi/dr = (1 + R^2/r^2)*u_inf*sintheta

Let's evaluate stagnation points, which is where V_r = 0 and V_theta = 0.

Vr=0 at r=R and at theta = pi/2 and 3pi/2.

V_theta = 0 when theta = 0 and pi.

So….

We get the stagnation points to be (R,0) and (R, pi)

There is a diagram of this on page 129

The final building block is a vortex

iv) It's on page 130

With all the building blocks in place, let's look at lifting flow over a cylinder.

Psi = u_inf(1 – R^2/r^2)*rsintheta = GAMMA/2pilnr/R

Finlnr/R is lnr – ln R – this will vanish with derivative so it doesn't affect the stream function. It's basically added in for convenience.

Finding V_r = 1/r*dpsi/dtheta = (1 – R^2/r^2) u_inf * costheta

V_theta = -dpsi/dr = -(1 + R^2/r^2) * u_inf * sintheta – GAMMA/2pir

If you evaluate at R = r, Vr = 0. And V_theta = -(1 + R^2/r^2) * uinf*sintheta – GAMA/2piR

This means that the surface of the cylinder is a streamline in and of itself.

= -uinf*sintheta-GAMMA/2piR

If we start evaluating at stagnation….

Singtheta = -GAMMA/4piRuinf.

Theta = sin(-1) (-GAMMA/4piRu_inf)

-This is telling is where the stagnation points should be

CASE 1 if GAMMA is less than 4piu_infR, which makes it less than 1 than the thing under the sin inverse is negative, so the stagnation points are in the 3rd and 4th quadrant. (diagram of thispage 132)

There is No symmetry about the xaxis, therefore we are producing Lift.

Symmetry about the y-axis, so no gdrag….but we can't have lift without drag. This is d'Alembert's paradox.

CASE 2…GAMMA = 4piu_infR

That means arcsin(-1) which is 3pi/2. That means we have 1 stagnation point, which is right at the bottom of the cylinder.

(diagram on page 132)

CASE 3, gamma is larger than 4piu_infR

This is non-physical.

CASE 4

If theta is pi/2 and 3pi/2, plug into Vr to get 0.

We get that r = GAMMA/4piu_inf + or minus sqrt(GAMMA/4piu_inf) – R^2)

The plus/minus means we have a non-physical stagnation point within the cylinder, but we also have a stangnation point outside and away from the bottom of the cylinder. (diagram on page 133)

Diagrams on page 133 of forces and angular momentums that fcorrespond to upwards and eownaward lift)

Kutta-Jokawski Theorem….

L' = rho_inf*u_inf*GAMMA (page 135)

This means lift per unit span is proportional to circulation.

These are valuable for low Re# for attached flow solutions. Low Re.

But at a higher Re, you have to go to N-S to solve for lift.

LET'S DO AIRFOILS NOW.

Starting with Thin airfoil Theory (page 136 has some nice pictures of starting vortex)

Kutta condition

-flow is going to end up leaving the top and bottom surfaces of an airfoil smoothly (following the curve of the airfoil.

-GAMMA is picked by nature such that this condition occurs.

(nice diagram on page 137)

Kelvin Circulation Theorem

The rat e of Change of circulation around a closed curve   consisting of the same fluid elements is equal to zero.

Kelvin;s Circulation theorem is demonstrated on page 138

Basically because of this theoreum we have a starting vortex and a clockwise circulation over the airfoil.

The clockwise circulation drives the stagnation point towards the TE.

DRAG is the mechanism that ensures that the Kutta condition is satisfied.

A NEW DAY

Kutta Joukowski is that:

L' = rho_in*v_inf*GAMMA

Kutta Condition

-Flow leaves the TE smoothly.

THIN AIRFOIL THEORY

What does "thin " mean?

-We are neglecting thickness effects.

-studying the effect of camber only

-Shape of camber line…z(x)

Slope is dz/dx

The free stream comes in at an angle to the camber line (diagram at 141)

Flow must be tangential to camber line at any point P on camber line.

Flow slope is w/u./.

w/u = dz/dx – alpha

u = v_inf

w/v_inf is about equal to dz/dx – alpha

w is about eaqual to dz/dx – alpha)

SO NOW Let's superimpose a building block to try to generate a lift.

Diagram on page 142

We get a vortex sheet of strength GAMMA.

And y(s) y is gamma) is the local vortex strength for the sheet

Vortex strength at ds: gamma(s)*ds.

Vortex

V_theta = -GAMMA/ 2pi*r)

Dw_p = -gamma(s)ds / 2pir

Because the airfoil is thin, the camber line isn't ar from the chord line.

Gamma(s) => y(x)

So let's put t he chord instead of the camber line.

ALSO IGNORING THE ROTIATION WE JUST MADE)

Page 143

Dxi is contributing to the vorticity at point P, so w(x) – integrate from 0 to C of gamma(xi)*d(..

Dxi is contributing to the vorticity at point P

(this is page 143)…we get the FUNAMENTAL EQUATION OF THIN AIRFOIL THEORY on page 144

Vn and w are normal to the camber line.

Therefore they cancel eachother out

IE.. the CAMBER LINE IS A STREAMLINE!

The only unknown in the equation is gamma(xi).

Let's look at a symmetric airfoil….

Dz/dx = 0.

So….

V_inf(alpha) = integral gamma(xi)d(xi) / 2pi(x – xi)

Let's change variables from here:

X = c/2(1-costheta0)

Xi = c/2(1- costheta)

Bounds of xi => 0 to C

New bonds:

Theta = 0 => costheta = 1

Xi = 0

Theta = pi => costheta = -1.

Xi = C

Dxi = c/2(sintheta) dtheta

Substitute into equations….

1/2pi from 0 to pi (gamma(theta) sinthetadtheta / costheta – costheta0 = v_inf (alpha – dz/dx)

About airfoils….the must satisfy the kutta condition.

What does that have to do with circulation strength?

(some derivation on page 145) the Local jump in tangential velocity across the vortex sheet is equal to the local vortex strength.

Gamma(TE) = V1 – V2 = 0.

The kutta condition

Assume a Fourier series solution to this problem….

Gamma(theta = 2V_inf [A0 1+costheta/sintheta + sum of Ansinntheta from n=1 to infinity]

Kutta Condition

TE = theta = pi

Our series solution checked out…

So… we can plug it in to the funcdamental thin airfoil equation:

Fundamental equation with series solution mapped.

IS ON PAGE 147.

A0 = alpha – 1/pi (dz/dxdethta0 (from 0 to pi)

An = 2/pi from 0 to pi integral dz/dxcosntheta0dtheta.

THESE POTENTIAL FLOW SOLUTIONS ARE ON PAGE 148

These are for page 147 and 148

For LIFT….

Derivations…derivations….

Coefficient of lift for thin airfoil theory for cambered airfoil….

L' and cl equations for lift involving series solution are on page 148 and 149!!

For a symmetric airfoil…

Cl is just 2pi*alpha

For cambered airfoil, cl = 2pi(alpha – alpha at l = 0)

Alpha at l=0 = -1/pi from 0 to pi dz/dx(cospi0 – 1) dtheta0

Let's look at pitching moment

The derivation for this is on page 150

For a symmetric airfoil…

Cm,c/4 = 0 at quarter chord and it is the center of pressure.

For symmetric airfoil, quarter-chord is the center of pressure.

If you want to computer center of pressure for cambered airfoil.

Cm,cp = cm,le + clxcp =0.

Xcp = ¼ [1 + pi/cl (A1 – A2)

This is the center of pressure for cambered airfoil.

AE3030

NEW NOTEBOOK

The first page is the fundamental equation for thin airfoil theory.

Basically it says that the camber lind is the stream line of the flow.

The first thing to solve is this equation (the fund eq) with a change of variables.

Xi = theta

X = theta0

Unknown becomes gamma(theta)

Assumed that there is a series solution for gamma

Assumed a series solution for gamma(theta)

We have a Fourier cosine series, A0, A1, A2

Cl, Cm A0, A1, A2

Cd = 0

Have to assume inviscid flow

-irrotational flow

Slope for Cl vs. alpha : 2pi. (alpha in radians)

Gow real is this data?

Page 345 of Andersen

In real life, the slope os a litter smaller than 2pi.

FINITE WING THEORY

3D finite span wings

--we get 3D flow effects

Wing tip vortices general spanwise components in the y-direction

Wingtip vortices downstream of the wing induce a small downward component of airvelocity in the neighborhood of the wing, itself called “downwash.”

Alpha_eff = effective aoa.

Alpha_ind = induced angle.

Vr = resultant V_inf.

Alpha = geometric angle.

So, alpha_eff is less than alpha.

This leads to a reduction of lift(diagrM IS ON PAGE 23 OF BOOK 2)

There is a component of local lift vector in the direction of V_inf. The pressure of downwash generaes DRAG.

It’s called induced drag.

We are still assuming incompressible, inviscid flow, but there is stll drag,.

W is called downwash velocity. This varies as a function of y. w = w(y) so ai = ai(y).

Induced drag:

I) 3D spanwise flow due to wingtip vortices alters the pressure distribution around the airfoil -➔ generates an imbalance in forces in v_inf direction.
II) ➔ induced drag
III) We can say it’s a type of pressure drag.

ii)

Wingtip vortices ➔ translational and rotational energies associated with them.

Energy comes from the aircraft ingine itself.

Basically not useful work.

TYPES OF DRAG

Di + df + dp.

Skin friction and separation drag are viscous effect derag “profile drag.”

What is /”profile drag” for potential flow analysis? 0.

Let’s talk VORTEX FILAMENT

-It is a straight LINE OF VORTEX STRENGTH gamma.

All pints vortex strength = GAMMA.

- Vortex sheet is made of filaments with constant GAMMA

Vortex filament extend in spanwise direction.

Vortex filaments can be curved.

How to find velocity at point P?

(diagram on page 5)

BIOT-SAVART LAW!

Dv = GAMMA/4pi (dlxr) / |r|^3

ANALOGOUS TO THE MAGNETIC FIELD STUFF.

That’s why we use the word “induced.”

There is a derivation to get the magnitude of velocity at point P from the vortex filemtn and it is on page 7.

|v| = GAMMA / 2pih, where h is the perpendicular distance to/pfrom the vortex filament.

Semi-infinite vortex filament

V = GAMMA/4pih

It's going from 0 to infinity instead of -in to inf.

HEMHOLTZ'S VORTEX THEOREM

i) Tehe strength of a vortex filament is constant along its length unless other vertices interact with it.
ii) Ii) a vortex filament cannot end in fluid. It must extend to the boundaries of the fluid or form a closed path.

Moving on to prandlt's classical lifting-line theory.

1911 -1918

Look at this derivation on page 8 and 9.

We get w(y) = -GAMMA/4pi * b/((b/2)^2 – y^2)

Problem with this is that it's no physical because we have these infinite lines at the tips.

So….we can consider multiple bound velocities:

Lifting line

-strength of each trailing vortex is equal to the change in circulation along the lifting line.

Infinite # of vortex filaments

(some more diagrams

(pick up from October 8^{th}, page 10 and 11)

Back to Lifting Lines

Let's consider multiple bound velocities:

The strength of each trailing vortex is equal to the change in circulation along the lifting lline

-Infinite number of vortex filaments

This is a trailing vortex sheet (page 10)

The total strength of the vortex sheet is 0. This is because all the vortex pairs dGAMMA, dGAMMMA cancel each other out.

The change in circulation along the lifting line: dGAMMA/dy * dy.

…is equal to the strength at the trailing vortex.

So What is w(y0) = -dGAMMA/dy * dy / 4pi (yo-y)

(The strength of the red line in the picture on page 11)

Downwash is

W(yo) = - integral from -b/2 to b/2 of dGAMMA/dy * dy / 4pi (yo-y)

(it's negative because it's downwash.

Now for some small angle hocus pocus….

You have an equation ofr induced angle of attack (page 12)

In terms of circulation

-it is a function of the span location.

Go back to the 2-D equation for Cl…

Cl = 2pi [alpha_eff(yo) – alphaL=0]

L' = rho_infv_infGAMMA(yo)

The non-dimensionalized version is…

L' = 1/2rho_infv_inf^2*c(yo) Cl

C is a function of yo if you have tapered wings or a geometric twist.

If the tip has a lower alpha it's called a "washout" and it's used for helicopter blades

If the tip has higher alpha, it's called a "wash in".

AERODYNAMIC TWIST

This means there is a different alphaL=0 for different airfoil sections.

Some part of the wing is NACA 0012, and some part is NCA 2412.

Equations for Cl coefficient of lift:

(they are on page 13)

Then you do some derivation and get the Fundamental Equation of Prandtl's Lifting Line Theory

Geometric angle of attack = effecting angle + induced angle

A NEW DAY AFTER THE BREAK

Finite Wing Theory (a review)

The strength of the trailing vortex and a nice picture is on page 15

Downwash at point yo tdue to the entire trailing vortex isheet is listed again on page 16.

induced angle of attack

ai(yo) = tan_1(-w(yo) / V_inf)

All this stuff is derived again on page 16-17.

Then we get to FUNDAMENTAL EQUATION OF PRANDTL'S LIFTING LINE THEORY (page 17)

Conceptually, it means that the angle of attack is equal to the effective attack plus the induced angle.

What can be obtained from this equation?

1) We can solve for lift distribution L'(yo) = rho_infv_infGAMMA(yo) if GAMMA(yo) is known
2) 2) we can solve for entire Lift
3) 3) we can get the lift coefficient for the entire wing
4) (These are all on page 17

INDUCED DRAG

(page 18)

Equations for it

Are on page 18.

ELLIPTICAL LIFT DISTRIBUTION

This is just an assumption we're gonna see if it fits in with the other stuff.

Derivation and equations are on pages 18 – 23

We get downwash, which isn't a function of span location because…for an ellipctically loaded wing, downwash is a CONSTANT

We also get angle of attack, coefficient of lift, induced drag and even Aerodynamic efficiency.

A review of Elliptical lift distribution is on page 27 going until page 29.

How can we achieve an elliptically loaded wing?

-No geometric twist

Alpha = const

-no aerodynamic twist (same airfoil throughout

CL = 2pi[alpha – alpha_i – alpha0]

-For an elliptically-loaded wing, alphai = also constant

CL(y) is therefore also constant as a function of span

(pictures of elliptically loaded wings on page 30)

GENERAL LIFT DISTRIBUTION.

These derivations and equations are on pages 31through 35.

Use the equation for alpha (that was on the slides) on page 38 to pick a theta.

Better look at the slides for the full equation.

HIGH LIFT DEVICES

A flap is a moveable appendage at the TE of the wing.

It increases camber.

-Fly slower during TO and landing so need more lift.

Pictures of these devices are on page 39.The ghigh lift devices explanations are on pages 39 through 42.